1.

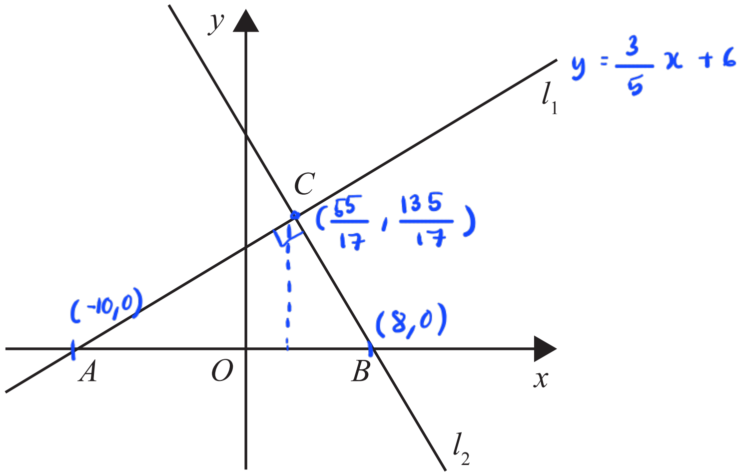


Figure 4

The line l_1 has equation $y = \frac{3}{5}x + 6$

The line l_2 is perpendicular to l_1 and passes through the point $B(8,0)$, as shown in the sketch in Figure 4.

(a) Show that an equation for line l_2 is

$$5x + 3y = 40$$

(3)

Given that

- lines l_1 and l_2 intersect at the point C
- line l_1 crosses the x-axis at the point A

(b) find the exact area of triangle ABC, giving your answer as a fully simplified fraction in the form $\frac{p}{q}$

(5)

a) l_1 : $y = \frac{3}{5}x + 6$, where gradient, $m_1 = \frac{3}{5}$

l_1 is perpendicular to l_2 , therefore $m_1 = \frac{-1}{m_2}$

$m_2 = -\frac{5}{3}$ (1)

Since l_2 passes through $B(8,0)$, we can find equation of l_2 :

$y - 0 = -\frac{5}{3}(x - 8)$ (1)

$y = \frac{-5x}{3} + \frac{40}{3} \implies 5x + 3y = 40$ (1)

b) Finding point A :

line l_1 cross the x-axis, so $y = 0$

$0 = \frac{3}{5}x + 6$

$x = -10 \quad \therefore A(-10, 0)$ 1

Finding point C :

solve equation l_1 and l_2 simultaneously to get point C .

$y = \frac{3}{5}x + 6$ — 1

$5x + 3y = 40$ — 2

Substitute 1 into 2

$5x + 3\left(\frac{3}{5}x + 6\right) = 40$ 1

$5x + \frac{9}{5}x + 18 = 40$

$\frac{34}{5}x = 22$

$34x = 110$

$x = \frac{55}{17}$ - substitute into 1

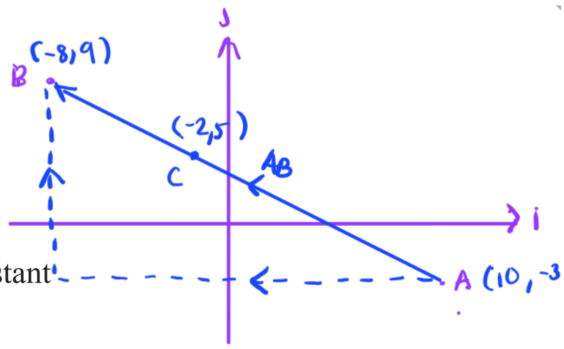
$y = \frac{3}{\cancel{5}}\left(\frac{\overset{11}{\cancel{55}}}{17}\right) + 6 = \frac{135}{17}$ 1 $\therefore C\left(\frac{55}{17}, \frac{135}{17}\right)$

length AB

height from x-axis to point C

Area of ABC : $\frac{1}{2} \times \text{length} \times \text{height} = \frac{1}{2} \times (8+10) \times \left(\frac{135}{17}\right)$ 1

$= \frac{1215}{17}$ 1



2. Relative to a fixed origin O

- point A has position vector $10\mathbf{i} - 3\mathbf{j}$
- point B has position vector $-8\mathbf{i} + 9\mathbf{j}$
- point C has position vector $-2\mathbf{i} + {}^{5}p\mathbf{j}$ where p is a constant.

(a) Find $\overrightarrow{AB}$

(2)

(b) Find $\left|\overrightarrow{AB}\right|$ giving your answer as a fully simplified surd.

(2)

$\frac{12}{18} \cdot \frac{2}{3}$

Given that points A, B and C lie on a straight line,

(c) (i) find the value of p,

(ii) state the ratio of the area of triangle AOC to the area of triangle AOB.

(3)

a) $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= -(10\underline{i} - 3\underline{j}) + (-8\underline{i} + 9\underline{j})$ (1)

$= -10\underline{i} - 8\underline{i} + 3\underline{j} + 9\underline{j}$

$= -18\underline{i} + 12\underline{j}$ (1)

b) $\left|\overrightarrow{AB}\right| = \sqrt{(-18)^2 + (12)^2}$

$= \sqrt{468}$ (1)

$= \sqrt{36} \times \sqrt{13}$

$= 6\sqrt{13}$ (1)

(c) (i) gradient BC = gradient BA (because all points are on the same line)

$$m_{BC} = \frac{9-p}{(-8)-(-2)} = \frac{9-p}{-6} \quad (1)$$

$$m_{BA} = \frac{9-(-3)}{(-8)-10} = \frac{12}{-18}$$

so, $\frac{9-p}{-6} = \frac{12}{-18}$

$$3(9-p) = 12$$
$$27 - 3p = 12$$
$$3p = 15$$
$$p = 5 \quad (1)$$

(ii) since length AC : AB is 2 : 3,

ratio of triangle AOC is 2:3 to triangle AOB. (1)

3. A circle has equation

$$x^2 + y^2 - 10x + 16y = 80$$

(a) Find

(i) the coordinates of the centre of the circle,

(ii) the radius of the circle.

(3)

Given that P is the point on the circle that is furthest away from the origin O,

(b) find the exact length OP

(2)

(a) (i) $x^2 + y^2 - 10x + 16y = 80$

'complete the square' on x and y terms.

$(x-5)^2 + (y+8)^2 - 5^2 - 8^2 = 80$

$(x-5)^2 + (y+8)^2 = 169$ 1

$x^2 + bx + c \downarrow (x + \frac{b}{2})^2 + c - (\frac{b}{2})^2$

$\therefore$ centre $= (5, -8)$ 1

A circle with centre (a, b) and radius r has equation: $(x-a)^2 + (y-b)^2 = r^2$

(ii) radius $= \sqrt{169}$
$= 13$ 1

(b) furthest point will be origin $\rightarrow$ centre + radius.

length $= \underbrace{\sqrt{5^2 + (-8)^2}}_{\text{origin to centre}} + \underbrace{13}_{\text{radius}}$ 1

$= \sqrt{89} + 13$ 1

"exact" so leave in this form.